

A Diagrammatic Approach to Crystalline Color Superconductivity

Jeffrey A. Bowers*, Joydip Kundu†, Krishna Rajagopal‡, Eugene Shuster§

*Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, MA 02139*

(MIT-CTP-3071, hep-ph/0101067, January 8, 2001.)

Abstract

We present a derivation of the gap equation for the crystalline color superconducting phase of QCD which begins from a one-loop Schwinger-Dyson equation written using a Nambu-Gorkov propagator modified to describe the spatially varying condensate. Some aspects of previous variational calculations become more straightforward when rephrased beginning from a diagrammatic starting point. This derivation also provides a natural base from which to generalize the analysis to include quark masses, nontrivial crystal structures, gluon propagation at asymptotic densities, and nonzero temperature. In this paper, we analyze the effects of nonzero temperature on the crystalline color superconducting phase.

*Email address: jbowers@mit.edu

†Email address: kundu@mit.edu

‡Email address: krishna@ctp.mit.edu

§Email address: eugeneus@mit.edu

I. INTRODUCTION AND SUMMARY

It is becoming widely accepted that at asymptotic densities the ground state of QCD with three massless quarks is the color-flavor locked (CFL) phase [1–3]. This phase features a condensate of Cooper pairs of quarks which includes ud , us and ds pairs. The CFL phase persists for finite masses, and even for unequal masses, so long as the differences are not too large [4,5]. It also persists in the presence of a nonzero electron chemical potential μ_e , so long as μ_e is not too large [6]. In the absence of any interaction (and thus in the absence of CFL pairing) either a quark mass difference or a nonzero μ_e pushes the Fermi momenta for different flavors apart, yielding different number densities for different flavors. In the CFL phase, however, the fact that the pairing energy is maximized when u , d and s number densities are equal enforces this equality [6]. This means that if one imagines increasing either the strange quark mass m_s or μ_e , nothing happens until a first order phase transition, at which CFL pairing is disrupted, (some) quark number densities spring free under the accumulated tension, and a less symmetric state of quark matter is obtained [6].

We can study much of the physics of interest by focussing just on pairing between massless up and down quarks with chemical potentials

$$\begin{aligned}\mu_u &= \bar{\mu} - \delta\mu \\ \mu_d &= \bar{\mu} + \delta\mu .\end{aligned}\tag{1}$$

For $0 < \delta\mu < \delta\mu_1$, the ground state is precisely that obtained for $\delta\mu = 0$ [7–9]. In this state, red and green up and down quarks pair, yielding four quasiparticles with superconducting gap Δ_0 [10–13]. And, the number density of red and green up quarks is the same as that of red and green down quarks. As $\delta\mu$ is increased from zero, this BCS state remains unchanged (and favored) because maintaining coincident Fermi surfaces maximizes the pairing and thus the gain in interaction energy. As $\delta\mu$ is increased further, the BCS state remains the ground state of the system only as long as its negative interaction energy offsets the large positive free energy cost associated with forcing the Fermi seas to deviate from their normal state distributions. In the weak coupling limit, in which $\Delta_0/\bar{\mu} \ll 1$, the BCS state persists while $\delta\mu < \delta\mu_1 = \Delta_0/\sqrt{2}$ [7,9].

For $\delta\mu \gg \delta\mu_1$, only very weak pairing between like-flavor quarks is possible [14]. Near the unpairing transition, however, another phase intervenes. This is the “LOFF” state, first explored by Larkin and Ovchinnikov and Fulde and Ferrell in the context of electron superconductivity in the presence of magnetic impurities [15,16]. Translating LOFF’s results to the case of interest, the authors of Ref. [9] found that for $\delta\mu \gtrsim \delta\mu_1$ it is favorable to form a state in which the u and d Fermi momenta are given by μ_u and μ_d as in the absence of interactions, and are thus not equal, but pairing nevertheless occurs. Whereas in the BCS state, obtained for $\delta\mu < \delta\mu_1$, pairing occurs between quarks with equal and opposite momenta, when $\delta\mu \gtrsim \delta\mu_1$ it is favorable to form a condensate of Cooper pairs with nonzero total momentum. This is favored because pairing quarks with momenta which are not equal and opposite gives rise to a region of phase space where each of the two quarks in a Cooper pair can be close to its Fermi surface, even when the up and down Fermi momenta

differ, and such pairs can be created at low cost in free energy.¹ Condensates of this sort spontaneously break translational and rotational invariance, leading to gaps which vary periodically in a crystalline pattern. If in some shell within the quark matter core of a neutron star (or within a strange quark star) the quark chemical potentials are such that crystalline color superconductivity arises, as occurs for a wide range of reasonable parameter values, rotational vortices may be pinned in this shell, making it a locus for glitch formation [9]. Rough estimates of the pinning force suggest that it is comparable to that for a rotational vortex pinned in the inner crust of a conventional neutron star, and thus may yield glitches of phenomenological interest [9].

The authors of Ref. [9] studied crystalline color superconductivity in a simplified model with two flavors of quarks with chemical potentials (1) which interact via a four-fermion interaction with the quantum numbers of single gluon exchange. In the LOFF state, each Cooper pair has total momentum $2\mathbf{q}$ with $|\mathbf{q}| \approx 1.2\delta\mu$. The direction of \mathbf{q} is chosen spontaneously. The LOFF phase is characterized by a gap parameter Δ and a diquark condensate, but not by an energy gap: the quasiparticle dispersion relations vary with the direction of the momentum, yielding gaps which vary from zero up to a maximum of Δ . The condensate is dominated by those regions in momentum space in which a quark pair with total momentum $2\mathbf{q}$ has both members of the pair within $\sim \Delta$ of their respective Fermi surfaces. The gap equation which determines Δ was derived in Ref. [9] using variational methods, along the lines of Refs. [16,19]. This gap equation can then be used to show that crystalline color superconductivity is favored over no ud pairing for $\delta\mu < \delta\mu_2$. Here, $\delta\mu_2 \approx 0.754\Delta_0$ if the coupling is weak [15,16,19] and if there is no interaction in angular momentum $J = 1$ channels [9]. For stronger coupling and for varying choices of interaction, $\delta\mu_2$ changes [9].

Crystalline color superconductivity is favored for $\delta\mu_1 < \delta\mu < \delta\mu_2$. As $\delta\mu$ increases, one finds a first order phase transition from the ordinary BCS phase to the crystalline color superconducting phase at $\delta\mu = \delta\mu_1$ and then a second order phase transition at $\delta\mu = \delta\mu_2$ at which Δ decreases to zero. Analysis of the Ginzburg-Landau effective potential which describes physics near $\delta\mu_2$ shows that $\Delta \sim (\delta\mu_2 - \delta\mu)^{1/2}$ for $\delta\mu \rightarrow \delta\mu_2$ [9]. Because the condensation energy in the LOFF phase is much smaller than that of the BCS condensate at $\delta\mu = 0$, the value of $\delta\mu_1$ is almost identical to that at which the naive unpairing transition from the BCS state to the state with no pairing would occur if one ignored the possibility of a LOFF phase. For all practical purposes, the LOFF gap equation is not required in order to determine $\delta\mu_1$. The LOFF gap equation is used to determine $\delta\mu_2$ and the properties of the LOFF phase [9]. For example, it determines the coefficients in the Ginzburg-Landau effective potential valid near $\delta\mu_2$ [19,9].

The variational derivation of the gap equation for the crystalline color superconducting phase is somewhat cumbersome [9]. One constructs a variational ansatz in which only

¹LOFF condensates have also recently been considered in two other contexts. In QCD with $\mu_u < 0$, $\mu_d > 0$ and $\mu_u = -\mu_d$, one has equal Fermi momenta for \bar{u} antiquarks and d quarks, BCS pairing between them, and consequently a $\langle \bar{u}d \rangle$ condensate [17]. If $-\mu_u$ and μ_d differ, and if the difference lies in the appropriate range, a LOFF phase with a spatially varying $\langle \bar{u}d \rangle$ condensate results [17]. Suitably isospin asymmetric nuclear matter may also admit LOFF pairing, as discussed recently in Ref. [18].

quarks within a “pairing region” are allowed to pair, minimizes the free energy with respect to all variational parameters (two per mode in momentum, color, flavor and spin space), and obtains a self-consistency relation which may then be solved to obtain Δ . The intricacy arises from the fact that the definition of the boundary of the pairing region involves Δ itself. A derivation in which one simply makes an ansatz for the quantum numbers of the condensate and then “turns a field-theoretical crank” and sees this intricate result emerge would be helpful both by virtue of being more straightforward and because the use of variational methods to obtain a gap equation is by now less familiar to many readers. We provide such a diagrammatic derivation here.

Furthermore, and as we explain at appropriate points in our presentation of the derivation in Sections II, III and IV, many generalizations are amenable to analysis using the formalism we present here:

- In Section V, we include the effects of nonzero temperature. We calculate the critical temperature T_c above which the crystalline color superconducting condensate vanishes and show that for $\delta\mu \rightarrow \delta\mu_2$, $T_c \rightarrow 0.39\Delta$, as previously known [19].
- It should also be straightforward to generalize the analysis to include three flavors of quarks with differing masses, and thus to study the crystalline color superconducting phases expected where either μ_e or m_s is just larger than that at which the CFL phase is lost [6]. This, not the toy model we analyze here, is the case of physical interest.
- As in Ref. [9], we restrict our attention here to the simplest possible “crystal” structure, namely that in which the condensate varies like a plane wave. Wherever this condensate is favored over the homogeneous BCS condensate and over the state with no pairing at all (i.e. where $\delta\mu_1 < \delta\mu < \delta\mu_2$) we expect that the true ground state of the system is a condensate which varies in space with some more complicated spatial dependence. The formalism we set up can be generalized to derive gap equations for, and hence to analyze and compare, condensates with arbitrary crystal structures in order to learn which one is favored.
- The diagrammatic analysis we present uses a point-like interaction between quarks, but the formalism is easily generalized to treat the exchange of a propagating gluon, as appropriate at asymptotically high densities. Even if such analyses, pioneered at $\delta\mu = 0$ in Ref. [20] and since studied in considerable detail by many authors [3], are to date of quantitative value only at inaccessibly high densities [21], it would be very interesting to see crystallization occurring in a controlled analysis beginning directly from the QCD Lagrangian.

II. THE GAP EQUATION FOR CRYSTALLINE COLOR SUPERCONDUCTIVITY

In the ordinary BCS phase, pairing between quarks with momentum \mathbf{p} and $-\mathbf{p}$ is described in the standard Nambu-Gorkov formalism by introducing an eight-component field

$$\Psi(p) = \begin{pmatrix} \psi(p) \\ \bar{\psi}^T(-p) \end{pmatrix}, \quad (2)$$

such that, in this basis, the inverse quark propagator takes the form

$$S^{-1}(p) = \begin{bmatrix} \not{p} + \mu\gamma_0 & \bar{\Delta}(p) \\ \Delta(p) & (\not{p} - \mu\gamma_0)^T \end{bmatrix}. \quad (3)$$

Here, $\bar{\Delta} = \gamma_0 \Delta^\dagger \gamma_0$ and $\bar{\Delta}$ is a matrix with color, flavor and Dirac indices which have all been suppressed. The diagonal blocks correspond to ordinary propagation and the off-diagonal blocks reflect the possibility of “anomalous propagation” in the presence of a diquark condensate $\langle \psi(\mathbf{x})\psi(\mathbf{x}) \rangle \propto \Delta$.

In the crystalline color superconducting phase [15,16,9], the condensate is made up of pairs of u and d quarks with momenta such that the total momentum of each Cooper pair is given by $2\mathbf{q}$ with $|\mathbf{q}| \approx 1.2\delta\mu$. The direction of \mathbf{q} is chosen spontaneously. Such a condensate varies periodically in space, with wavelength $\pi/|\mathbf{q}|$:

$$\langle \psi(\mathbf{x})\psi(\mathbf{x}) \rangle \propto \Delta e^{2i\mathbf{q}\cdot\mathbf{x}}. \quad (4)$$

The spatial dependence (4) is only the simplest possible choice. Wherever (4) is favored over both the ordinary BCS state and the state with no pairing at all, we expect that the true ground state of the system will include Cooper pairs with their respective $|\mathbf{q}|$ ’s taking on the same, energetically favored, value, but choosing one of several spontaneously selected directions. The result would be a condensate which varies in space like a sum of plane waves. For example, a cubic crystal arises as a sum of six plane waves. The favored crystal structure for the crystalline color superconductor is not known. In this paper, as in Ref. [9], we only consider the simplest possibility (4).

Although we expect that crystalline color superconductivity occurs whenever the mass difference or chemical potential difference between any two flavors of quarks is just larger than the maximum value which the standard BCS state can tolerate, for concreteness we shall follow Ref. [9] and only consider pairing between massless u and d quarks with chemical potentials (1). In the condensate (4), u quarks with momentum $\mathbf{p} + \mathbf{q}$ pair with d quarks with momentum $-\mathbf{p} + \mathbf{q}$. To describe this, we must use a modified Nambu-Gorkov spinor defined as

$$\Psi(p, q) = \begin{pmatrix} \psi_u(p + q) \\ \psi_d(p - q) \\ \bar{\psi}_d^T(-p + q) \\ \bar{\psi}_u^T(-p - q) \end{pmatrix}. \quad (5)$$

Note that flavor indices are now explicit, which will be convenient below. The central change we have made in going from (2) to (5) is to modify the momentum dependence. Note that by q we mean the four-vector $(0, \mathbf{q})$. The Cooper pairs have nonzero total momentum, but the ground state condensate (4) is static. The change from (2) to (5) can be seen as a change of basis. In the presence of a crystalline color superconducting condensate, anomalous propagation does not only mean picking up or losing two quarks from the condensate. It also means picking up or losing momentum $2\mathbf{q}$. If we tried to describe this using the original basis (2), the inverse quark propagator would no longer be diagonal in momentum space. The new basis (5) has been chosen so that the inverse quark propagator in the crystalline color superconducting phase is diagonal in p -space and is given by

$$S^{-1}(p, q) = \begin{bmatrix} \not{p} + \not{q} + \mu_u \gamma_0 & 0 & -\bar{\Delta}(p, -q) & 0 \\ 0 & \not{p} - \not{q} + \mu_d \gamma_0 & 0 & \bar{\Delta}(p, q) \\ -\Delta(p, -q) & 0 & (\not{p} - \not{q} - \mu_d \gamma_0)^T & 0 \\ 0 & \Delta(p, q) & 0 & (\not{p} + \not{q} - \mu_u \gamma_0)^T \end{bmatrix}. \quad (6)$$

$2\mathbf{p}$ is the relative momentum of the quarks in a given pair, and is different for different pairs. In the gap equation below, we shall integrate over p_0 and \mathbf{p} , as we sum the contribution of all pairs. $2\mathbf{q}$ is the center of mass momentum of every pair in the condensate; it is a constant and thus will not be integrated over. It is convenient to denote flavor indices explicitly in (6) because we are describing the situation where $\mu_u \neq \mu_d$. It is straightforward to introduce different quark masses in Eq. (6), but then the calculations become more involved and we therefore defer this to a future publication. Note that the condensate is explicitly antisymmetric in flavor. Color and Dirac indices remain suppressed. As desired, the off-diagonal blocks describe anomalous propagation in the presence of a condensate of diquarks with momentum $2\mathbf{q}$. The choice of basis we have made is analogous to that introduced previously in the analysis of a crystalline quark-antiquark condensate [22]. This work also points the way toward the generalization of (5) needed to handle a condensate which varies in space like a cubic crystal rather than the plane wave (4).

We wish to obtain the gap by solving the one-loop Schwinger-Dyson equation, given by

$$S^{-1}(k, q) - S_0^{-1}(k, q) = ig^2 \int \frac{d^4 p}{(2\pi)^4} \Gamma_\mu^A S(p, q) \Gamma_\nu^B D_{AB}^{\mu\nu}(k - p). \quad (7)$$

Here, $D_{AB}^{\mu\nu}$ is the gluon propagator, S is the full quark propagator, whose inverse is given by (6), and S_0 is the fermion propagator in the absence of interaction, given by S with $\Delta = 0$. S_0 looks unusual, because it depends on both k and the “offset” \mathbf{q} . This is a consequence of our choice of basis (5), and would be a legitimate if perverse way to describe noninteracting fermions. This choice of basis is natural in the crystalline color superconducting phase. The vertices are defined as follows:

$$\Gamma_\mu^A = \begin{pmatrix} \gamma_\mu \lambda^A / 2 & 0 & 0 & 0 \\ 0 & \gamma_\mu \lambda^A / 2 & 0 & 0 \\ 0 & 0 & -(\gamma_\mu \lambda^A / 2)^T & 0 \\ 0 & 0 & 0 & -(\gamma_\mu \lambda^A / 2)^T \end{pmatrix}. \quad (8)$$

Note that in Eq. (7) we have chosen to work in Minkowski space. We will continue to work in Minkowski space until we obtain the gap equation itself, which we will then write in Euclidean space for computational convenience.

We defer the analysis of the crystalline color superconducting phase at asymptotically high densities to future work. In this paper, as in Ref. [9], we choose to caricature the interaction between quarks as a point-like four-fermion interaction with the quantum numbers of single-gluon exchange. This means that in (7), we make the replacement

$$g^2 D_{AB}^{\mu\nu} \rightarrow -3G g^{\mu\nu} \delta_{AB} \quad (9)$$

where G , normalized as in Ref. [3], is a dimensionful coupling constant which parametrizes the strength of the interaction between quarks. Reasonable choices for G , motivated by zero

density hadron phenomenology, yield a BCS gap on the order of 100 MeV at $\bar{\mu} = 400$ MeV in the absence of any chemical potential difference $\delta\mu$ [3].

Once we have removed the gluon propagator from (7), we see that the right-hand side is independent of k . The left-hand side must therefore be independent of k as well, meaning that Δ in (6) must be independent of p . Δ does depend on the common momentum of all the Cooper pairs, $2\mathbf{q}$. Choosing an ansatz for Δ is straightforward once we have understood that it must be independent of p . Single gluon exchange is attractive in the color-antisymmetric ($\bar{\mathbf{3}}$), flavor antisymmetric, Lorentz scalar or pseudoscalar channels. Instanton effects favor the scalar condensate, and we therefore make the ansatz

$$\Delta^{\alpha\beta}(p, q) = \epsilon^{\alpha\beta 3} C \gamma_5 \Delta \quad (10)$$

for the gap matrix, where α and β are color indices, running from 1 to 3, and $C = i\gamma_0\gamma_2$. Δ has no remaining indices and all the matrix structure has now been written explicitly. Δ does depend on $|\mathbf{q}|$, although we do not denote this dependence explicitly, but does not depend on p or the direction of \mathbf{q} .

After some algebra (essentially the determination of S given S^{-1} specified above), and upon suitable projection, the Schwinger-Dyson equation (7) reduces to a gap equation for the gap parameter Δ given (in Euclidean space) by

$$\Delta = 2G \int \frac{d^4p}{(2\pi)^4} \frac{4\Delta w}{w^2 - 4[(|\mathbf{p}|^2 - (ip_0 + \delta\mu)^2)(\bar{\mu}^2 - |\mathbf{q}|^2) + (\mathbf{p} \cdot \mathbf{q} + \bar{\mu}(ip_0 + \delta\mu))^2]} \quad (11)$$

where $w = |\mathbf{p}|^2 - |\mathbf{q}|^2 - (ip_0 + \delta\mu)^2 + \bar{\mu}^2 + \Delta^2$.

We analyze the gap equation (11) in Section IV. It will turn out to be close to, but not identical to, that derived in Ref. [9]. The difference is that here we have kept the contributions of particles, holes, and antiparticles in the gap equation, whereas in Ref. [9] only particle-particle and hole-hole pairing was considered. Pairing in the crystalline color superconducting phase is dominated by those pairs in which both particles or both holes in a pair are near their respective Fermi surfaces. Indeed, it is the fact that such pairs exist even at nonzero $\delta\mu$ as long as $|\mathbf{q}| \geq \delta\mu$ which explains why the crystalline color superconducting phase may be favored in the first place. We therefore expect that neglecting the contributions of the antiparticles in the gap equation, as was done in Ref. [9], should be a good approximation. Demonstrating this requires complicating the gap equation considerably at first, although it does eventually simplify as will be shown by the end of Section III. We shall see in Section IV that once the contributions of antiparticles have been eliminated, the gap equation we derive here agrees with that of Ref. [9].

III. ELIMINATING ANTIPARTICLES

In order to eliminate the (small) contribution of the antiparticles on the right-hand side of the gap equation, we shall need the projectors [23,24]

$$\begin{aligned} P_+(p) &= \frac{1 + \vec{\alpha} \cdot \hat{p}}{2} \\ P_-(p) &= \frac{1 - \vec{\alpha} \cdot \hat{p}}{2} \end{aligned} \quad (12)$$

with $\vec{\alpha} = \gamma_0 \vec{\gamma}$, where P_+ projects onto particle states and P_- projects onto antiparticle states. This allows us to separate the ansatz for Δ into those parts which include antiparticle pairing and those which do not. We replace the ansatz (10) by

$$\Delta^{\alpha\beta}(p, -q) = \epsilon^{\alpha\beta 3} C \gamma^5 \left[\Delta_1 P_+(p-q) P_+(p+q) + \Delta_2 P_-(p-q) P_-(p+q) + \Delta_3 P_+(p-q) P_-(p+q) + \Delta_4 P_-(p-q) P_+(p+q) \right]. \quad (13)$$

Here, $\Delta_{1,2,3,4}$ are four (potentially different) gap parameters whose meaning we now explain. To understand each of the terms in (13), note that, for example,

$$C \gamma^5 P_+(p-q) P_+(p+q) = P_+^T(-p+q) C \gamma^5 P_+(p+q).$$

Thus, Δ_1 describes pairing between particles (and not antiparticles) with momenta $\mathbf{p} + \mathbf{q}$ and $-\mathbf{p} + \mathbf{q}$. Similarly, Δ_2 describes antiparticle–antiparticle pairing, and Δ_3 and Δ_4 describe particle–antiparticle pairing, which is only possible for $\mathbf{q} \neq 0$. (For $\mathbf{q} = 0$, the only projectors which occur are $P_+(p)$ and $P_-(p)$, and $P_+(p)P_-(p) = 0$.)

With our point-like interaction, Δ must be independent of p . This requires

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 \equiv \Delta, \quad (14)$$

which restores the simple ansatz (10). It may seem perverse, but we now derive coupled gap equations for $\Delta_{1,2,3,4}$, without assuming that they are equal. We do so for two reasons. First, in a future publication, we plan to restore the gluon propagator. In this context, Δ is not independent of p and $\Delta_{1,2,3,4}$ therefore need not all be the same. The exercise below therefore lays the groundwork for this future calculation. Second, and in the present context, we wish to eliminate all terms on the right-hand side of the gap equation which depend on $\Delta_{2,3,4}$, as they make only a small contribution. The reader not interested in details of this derivation can safely skip to Eqs. (21) and (22) and the discussion that follows them.

The Schwinger-Dyson equation (7) (using Eq. (9)) is an equation for the matrix Δ :

$$\Delta(k, q) = 3iG \int \frac{d^4 p}{(2\pi)^4} \left(\gamma_\mu \frac{\lambda^A}{2} \right)^T S_{42}(p, q) \left(\gamma^\mu \frac{\lambda^A}{2} \right) \quad (15)$$

where $S_{42}(k, q)$ is the (4,2)-component of the fermion propagator found by inverting Eq. (6). After some algebra, we find

$$S_{42}(p, q) = - \left(\not{p} + \not{q} - \mu_u \gamma^0 \right)^{-1T} \Delta(p, q) \left[\not{p} - \not{q} + \mu_d \gamma^0 - \bar{\Delta}(p, q) \left(\not{p} + \not{q} - \mu_u \gamma^0 \right)^{-1T} \Delta(p, q) \right]^{-1}. \quad (16)$$

Upon inserting the ansatz (13) for $\Delta(p, q)$, we can rewrite (16) in terms of the gap parameters $\Delta_{1,2,3,4}$. In order to display the resulting expressions, we must first define:

$$\begin{aligned} A &= p_0 + \mu_d - |\mathbf{p} - \mathbf{q}| - \Delta_1 \frac{\Delta_1 \sin^2(\beta/2) + 1/2 \Delta_3 \cos \beta}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} - \Delta_4 \frac{\Delta_4 \cos^2(\beta/2) - 1/2 \Delta_2 \cos \beta}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} \\ B &= p_0 + \mu_d + |\mathbf{p} - \mathbf{q}| - \Delta_2 \frac{\Delta_2 \sin^2(\beta/2) + 1/2 \Delta_4 \cos \beta}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} - \Delta_3 \frac{\Delta_3 \cos^2(\beta/2) - 1/2 \Delta_1 \cos \beta}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} \\ C &= -\frac{1}{2} \left(\frac{\Delta_1 \Delta_3}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} - \frac{\Delta_2 \Delta_4}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} \right). \end{aligned} \quad (17)$$

Here, the angle β is defined as the angle between the up quark momentum $\mathbf{q} + \mathbf{p}$ and the down quark momentum $\mathbf{q} - \mathbf{p}$ and is therefore given by

$$\cos \beta = (\widehat{\mathbf{q} + \mathbf{p}}) \cdot (\widehat{\mathbf{q} - \mathbf{p}}) . \quad (18)$$

With these definitions, $S_{42}(p, q)$ becomes

$$S_{42}(p, q) = \epsilon^{\alpha\beta 3} C \gamma_5 T(p, q) \quad (19)$$

with

$$\begin{aligned} T(p, q) = & - \left[\frac{B}{AB - C^2 + (A - B)C \cos \beta} \left(\frac{\Delta_1}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} P_-(p + q) P_-(p - q) \right. \right. \\ & \left. \left. + \frac{\Delta_4}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} P_+(p + q) P_-(p - q) \right) \right. \\ & + \frac{A}{AB - C^2 + (A - B)C \cos \beta} \left(\frac{\Delta_3}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} P_-(p + q) P_+(p - q) \right. \\ & \left. + \frac{\Delta_2}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} P_+(p + q) P_+(p - q) \right) \\ & + \frac{C}{AB - C^2 + (A - B)C \cos \beta} \left(\frac{\Delta_3 - \Delta_1}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} P_-(p + q) P_+(p - q) P_+(p + q) \right. \\ & + \frac{\Delta_4 - \Delta_2}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} P_+(p + q) P_+(p - q) P_-(p + q) \\ & \left. \left. - \frac{\Delta_1 \sin^2 \frac{\beta}{2} + \Delta_3 \cos^2 \frac{\beta}{2}}{p_0 - \mu_u + |\mathbf{p} + \mathbf{q}|} P_-(p + q) + \frac{\Delta_2 \sin^2 \frac{\beta}{2} + \Delta_4 \cos^2 \frac{\beta}{2}}{p_0 - \mu_u - |\mathbf{p} + \mathbf{q}|} P_+(p + q) \right) \right] . \quad (20) \end{aligned}$$

Noting that

$$\left(\frac{\lambda^A}{2} \right)^T \lambda^2 \left(\frac{\lambda^A}{2} \right) = -\frac{2}{3} \lambda^2 ,$$

we obtain the gap equation

$$\begin{aligned} & \Delta_1 P_+(k + q) P_+(k - q) + \Delta_2 P_-(k + q) P_-(k - q) \\ & + \Delta_3 P_+(k + q) P_-(k - q) + \Delta_4 P_-(k + q) P_+(k - q) = -2iG \int \frac{d^4 p}{(2\pi)^4} \gamma_\mu T(p, q) \gamma^\mu . \quad (21) \end{aligned}$$

Upon setting all the Δ 's equal as in (14), the gap equation (21) yields

$$\Delta = -2iG \int \frac{d^4 p}{(2\pi)^4} \text{Tr} T(p, q) . \quad (22)$$

When written explicitly, this is Eq. (11).

Using the definition of $T(p, q)$, we can easily identify the contributions of the four Δ 's to the right-hand side of the gap equation. Since we expect that the Δ_1 terms, which describe particle-particle and hole-hole pairing, will give the dominant contributions to the gap integral, we now eliminate all terms in $T(p, q)$ which depend on $\Delta_{2,3,4}$. This means

that the contribution of the antiparticles to the right-hand side of the gap equation has been eliminated. Note that we are *not* setting $\Delta_{2,3,4} = 0$. With a point-like interaction, all the Δ 's are in fact equal as in (14), and the left-hand side of the gap equation (21) is k -independent and equal to Δ . The point is that the contributions of those terms on the right-hand side of (21) in which $\Delta_{2,3,4}$ appear must be small, and we can therefore neglect them. In other words, once all integrations have been completed on the right-hand side, we would find that $\Delta_{2,3,4}$ only occur multiplied by quantities which are small if Δ/μ is small. Dropping $\Delta_{2,3,4}$ on the right-hand side before integration (but keeping them on the left-hand side) should therefore be a good approximation. The resulting gap equation can be written (in Euclidean space) as

$$\Delta = 2G \int \frac{d^4 p}{(2\pi)^4} \frac{2\Delta \sin^2 \frac{\beta}{2}}{(p_0 - iE_1(\mathbf{p}))(p_0 + iE_2(\mathbf{p}))} \quad (23)$$

where $E_{1,2}(\mathbf{p})$ are defined as in Ref. [9]:

$$\begin{aligned} E_1(\mathbf{p}) &= +\delta\mu + \frac{1}{2}(|\mathbf{p} + \mathbf{q}| - |\mathbf{p} - \mathbf{q}|) + \frac{1}{2}\sqrt{(|\mathbf{p} + \mathbf{q}| + |\mathbf{p} - \mathbf{q}| - 2\bar{\mu})^2 + 4\Delta^2 \sin^2 \frac{\beta}{2}} \\ E_2(\mathbf{p}) &= -\delta\mu - \frac{1}{2}(|\mathbf{p} + \mathbf{q}| - |\mathbf{p} - \mathbf{q}|) + \frac{1}{2}\sqrt{(|\mathbf{p} + \mathbf{q}| + |\mathbf{p} - \mathbf{q}| - 2\bar{\mu})^2 + 4\Delta^2 \sin^2 \frac{\beta}{2}} \end{aligned} \quad (24)$$

and β is defined in Eq. (18). As we describe below, we have confirmed explicitly that the gap equation (23) is a good approximation to Eq. (11).

IV. ANALYZING THE GAP EQUATION

The energies E_1 and E_2 given in (24) arise in Ref. [9]. There, we deduced that the right-hand side of the gap equation must be taken to vanish in those regions of \mathbf{p} -space where either $E_1(\mathbf{p})$ or $E_2(\mathbf{p})$ is negative via the following argument. In the region where $E_1(\mathbf{p}) < 0$, it is free-energetically favorable to have unpaired u -quarks rather than pairs. Similarly, in the region where $E_2(\mathbf{p}) < 0$, it is free-energetically favorable to have unpaired d -quarks rather than pairs. Because quarks do not pair in these “blocking regions” of momentum space, these regions do not contribute to the gap equation, which becomes an integral over those regions of momentum space wherein pairing occurs. If Δ is set to zero, the blocking regions are simply described. They are the regions in \mathbf{p} -space where the u -quark state with momentum $\mathbf{p} + \mathbf{q}$ is within the u Fermi sea while the d -quark state with momentum $-\mathbf{p} + \mathbf{q}$ is outside the d Fermi sea, or vice versa. In the presence of a nonzero Δ , the boundaries of the blocking regions are given by $E_1(\mathbf{p}) = 0$ and $E_2(\mathbf{p}) = 0$ and therefore depend on Δ and are not simply determined by the locations of the noninteracting Fermi surfaces. The result of this analysis, presented in Ref. [9], is a variational procedure in which the boundaries of the blocking regions, and thus the specification of the variational ansatz itself, depend on the gap Δ , which is in turn obtained by solving a gap equation whose integrand is restricted by hand to vanish within said blocking regions.

In contrast to the intricacy of the variational approach, the physics of the blocking regions emerges from a completely straightforward analysis of the gap equation in the form we have derived above, namely Eq. (23). We simply do the p_0 integral by contour integration. There are two poles, both of which lie on the imaginary axis. Let us close the contour in the upper

half plane. If $E_1 > 0$ and $E_2 > 0$, we pick up the pole at $p_0 = iE_1$ which has a residue proportional to $1/(E_1 + E_2)$. If $E_1 < 0$ and $E_2 > 0$, both poles are in the lower half plane, and the right-hand side of the gap equation vanishes. If $E_1 > 0$ and $E_2 < 0$, both poles are in the upper half plane, the residues from the two poles cancel, and the right-hand side of the gap equation again vanishes. (If we close the contour in the lower half plane, we obtain the same result upon noticing that we encircle no poles if $E_1 < 0$ and $E_2 > 0$ and two poles with cancelling residues for $E_1 > 0$ and $E_2 < 0$.) Thus, upon doing the p_0 integration we obtain the gap equation of Ref. [9]:

$$\begin{aligned} 1 &= 2G \int_{\mathbf{p} \in \mathcal{P}} \frac{d^3p}{(2\pi)^3} \frac{2 \sin^2 \frac{\beta}{2}}{E_1(\mathbf{p}) + E_2(\mathbf{p})} \\ &= 2G \int_{\mathbf{p} \in \mathcal{P}} \frac{d^3p}{(2\pi)^3} \frac{2 \sin^2 \frac{\beta}{2}}{\sqrt{(|\mathbf{p} + \mathbf{q}| + |\mathbf{p} - \mathbf{q}| - 2\bar{\mu})^2 + 4\Delta^2 \sin^2 \frac{\beta}{2}}} \end{aligned} \quad (25)$$

where the “pairing region” \mathcal{P} in \mathbf{p} -space is given by

$$\mathcal{P} = \{\mathbf{p} \mid E_1(\mathbf{p}) > 0 \text{ and } E_2(\mathbf{p}) > 0\} . \quad (26)$$

Thus, a trivial exercise in residue calculus has reproduced the blocking regions, excluding from the gap equation those regions in momentum space where $E_1(\mathbf{p})$ or $E_2(\mathbf{p})$ is negative. Note that because $E_1(\mathbf{p}) + E_2(\mathbf{p}) \geq 0$, as can be seen from the definitions (24), there is no value of \mathbf{p} for which both E_1 and E_2 are negative. Note also that the gap equation is dominated by those regions in momentum space where $E_1(\mathbf{p}) + E_2(\mathbf{p})$ is as small as possible, where the integrand in (25) is of order $1/\Delta$. These values of \mathbf{p} are such that both members of a LOFF pair have momenta close to (within $\sim \Delta$ of) their respective Fermi surfaces. That is, $|\mathbf{p} + \mathbf{q}|$ is within Δ of μ_u and $|\mathbf{p} - \mathbf{q}|$ is within Δ of μ_d .

For completeness, we sketch the analysis of (11), in which the contributions of antiparticle pairing to the gap equation have not been eliminated. The denominator of the integrand in (11) is a fourth order polynomial in p_0 , so the gap equation can be rewritten as

$$\Delta = 2G \int \frac{d^4p}{(2\pi)^4} \frac{4\Delta w}{(p_0 - iP_1(\mathbf{p}))(p_0 + iP_2(\mathbf{p}))(p_0 + i\bar{P}_1(\mathbf{p}))(p_0 - i\bar{P}_2(\mathbf{p}))} . \quad (27)$$

The analytical expressions for the poles $P_1(\mathbf{p})$, $P_2(\mathbf{p})$, $\bar{P}_1(\mathbf{p})$, and $\bar{P}_2(\mathbf{p})$ are complicated and uninformative. However, we have checked that for reasonable choices of parameters, the numerical values of $P_1(\mathbf{p})$ and $P_2(\mathbf{p})$ are very close to $E_1(\mathbf{p})$ and $E_2(\mathbf{p})$ and those of $\bar{P}_1(\mathbf{p})$ and $\bar{P}_2(\mathbf{p})$ are very close to the antiparticle energies

$$\begin{aligned} \bar{E}_1(\mathbf{p}) &= -\delta\mu + \frac{1}{2}(|\mathbf{p} + \mathbf{q}| - |\mathbf{p} - \mathbf{q}|) + \frac{1}{2}\sqrt{(|\mathbf{p} + \mathbf{q}| + |\mathbf{p} - \mathbf{q}| + 2\bar{\mu})^2 + 4\Delta^2 \sin^2 \frac{\beta}{2}} \\ \bar{E}_2(\mathbf{p}) &= +\delta\mu - \frac{1}{2}(|\mathbf{p} + \mathbf{q}| - |\mathbf{p} - \mathbf{q}|) + \frac{1}{2}\sqrt{(|\mathbf{p} + \mathbf{q}| + |\mathbf{p} - \mathbf{q}| + 2\bar{\mu})^2 + 4\Delta^2 \sin^2 \frac{\beta}{2}} . \end{aligned} \quad (28)$$

The analysis of (27) is analogous to that of (23). Wherever both P_1 and P_2 are positive, the pole at $p_0 = iP_1$ contributes, with residue proportional to $1/(P_1 + P_2)$. The integral is dominated by the region where $P_1 + P_2$ is close to zero. There is also a contribution from the pole at $p_0 = i\bar{P}_2$, but the residue of this pole is nowhere large. For this reason,

and because P_1 and P_2 are numerically very close to E_1 and E_2 , we find that (23), and thus (25) which was derived variationally in Ref. [9], is a good approximation to (27). For $\bar{\mu} = 400$ MeV and G chosen such that the BCS gap at $\delta\mu = 0$ is 100 MeV, we take $\delta\mu$ within the range where the crystalline color superconducting phase is favored [9] and choose a value of Δ which solves (23). We then find that the right-hand sides of (23) and (27) differ by about 20%. The discrepancy vanishes in the weak-coupling limit. (Note that eliminating the contribution of the pole at $p_0 = i\bar{P}_2$ does not by itself reduce (27) to (23), although it does change the 20% discrepancy to a 1% discrepancy. Eliminating all the contributions of the antiparticles is more subtle, as we have seen.)

Although the expressions for the $P(\mathbf{p})$'s are much more complicated than those for the $E(\mathbf{p})$'s, the gap equation (27) actually turns out to be more easily solvable (by Mathematica) than (23), as we now explain. The blocking regions in (27) are regions wherein either P_1 or P_2 is negative, and are therefore bounded by surfaces on which P_1 or P_2 vanishes. Within these regions, there is no contribution from the P_1 and P_2 poles. The simplification which occurs in (27) is in the explicit expressions for the boundaries of the blocking regions. We simply set $p_0 = 0$ in the denominator of Eq. (11) and for each value of $|\mathbf{p}|$ we solve for $\cos\theta$, the angle between \mathbf{p} and \mathbf{q} :

$$\cos\theta = -\frac{2\bar{\mu}\delta\mu \pm \sqrt{(|\mathbf{p}|^2 - |\mathbf{q}|^2 - \delta\mu^2 + \bar{\mu}^2 + \Delta^2)^2 - 4(|\mathbf{p}|^2 - \delta\mu^2)(\bar{\mu}^2 - |\mathbf{q}|^2)}}{2|\mathbf{p}||\mathbf{q}|}. \quad (29)$$

This allows one to implement the fact that the \mathbf{p} -integral is to be taken over all of \mathbf{p} -space except for the blocking regions via explicitly specified limits on the $\cos\theta$ integral. In contrast, even though E_1 and E_2 are simpler than P_1 and P_2 , the dependence of the denominator in (23) on $\cos\theta$ is more complicated than that in (11), and the blocking regions can only be specified explicitly as roots of a quartic polynomial.

The calculation of $\delta\mu_2$ of Ref. [9] follows directly from Eq. (23), as does the value of $|\mathbf{q}|$, which is given to a very good approximation by that for $\delta\mu \rightarrow \delta\mu_2$. (One finds $\delta\mu_2$ by seeking the largest value of $\delta\mu$ for which there is a choice of $|\mathbf{q}|$ which yields a nonzero solution Δ to the gap equation (23) [9].) Our rederivation of (23) has several merits. First, as it begins with a Schwinger-Dyson equation rather than a variational wave function, it may appear more familiar. Second, the emergence of blocking regions is straightforward. Third, it is the basis for many generalizations: Quark masses can easily be introduced in (6), and the analogue of (11) can then be derived. The gluon propagator need not be replaced by a point-like interaction. This opens the way to a treatment of color superconductivity at asymptotically high density. Also, nontrivial crystal structures can be analyzed beginning with a Nambu-Gorkov propagator which admits “anomalous propagation” in which $2\mathbf{q}$ of momentum is gained or lost, for several different values of the vector $2\mathbf{q}$. Finally, the generalization to nonzero temperature is straightforward. To this we now turn.

V. CRYSTALLINE COLOR SUPERCONDUCTIVITY AT NONZERO TEMPERATURE

We can now derive the gap equation for the LOFF state at nonzero temperature. We begin with Eq. (23). Using the standard formalism, we obtain the nonzero temperature gap

equation by converting the p_0 integral into a sum over Matsubara frequencies. That is, with $\omega_n = (2n + 1)\pi T$, using the prescription

$$p_0 \rightarrow \omega_n \quad \text{and} \quad \int \frac{dp_0}{2\pi} \rightarrow T \sum_n$$

we obtain the following equation:

$$\Delta = 2G \int \frac{d^3 p}{(2\pi)^3} 2\Delta \sin^2 \left(\frac{\beta}{2} \right) T \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n - iE_1(\mathbf{p}))(\omega_n + iE_2(\mathbf{p}))} . \quad (30)$$

The sum may be evaluated by converting it into a contour integral:

$$\sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n - iE_1)(\omega_n + iE_2)} = \frac{1}{2\pi i} \int_C dz \frac{1}{T(e^{z/T} + 1)} \frac{1}{(z + E_1)(z - E_2)}$$

where the contour C encircles the imaginary axis.² We may deform the contour so that it encircles, with negative orientation, the poles off the imaginary axis. This gives

$$\begin{aligned} \frac{1}{2\pi i} \int_C dz \frac{1}{T(e^{z/T} + 1)} \frac{1}{(z + E_1)(z - E_2)} &= -\frac{1}{T(e^{E_2/T} + 1)} \frac{1}{E_2 + E_1} + \frac{1}{T(e^{-E_1/T} + 1)} \frac{1}{E_2 + E_1} \\ &= \frac{1}{2T(E_1 + E_2)} \left[\tanh \left(\frac{E_1}{2T} \right) + \tanh \left(\frac{E_2}{2T} \right) \right] . \end{aligned}$$

Upon using these identities in the finite-temperature gap equation Eq. (30), we obtain:

$$1 = 2G \int \frac{d^3 p}{(2\pi)^3} \frac{2 \sin^2 \left(\frac{\beta}{2} \right)}{E_1(\mathbf{p}) + E_2(\mathbf{p})} \frac{1}{2} \left[\tanh \left(\frac{E_1(\mathbf{p})}{2T} \right) + \tanh \left(\frac{E_2(\mathbf{p})}{2T} \right) \right] . \quad (31)$$

Note that here the integration is performed over all of \mathbf{p} -space: there are no blocking regions at nonzero temperature. The blocking regions emerge in the limit $T \rightarrow 0$ as follows: if $E_1(\mathbf{p}) > 0$ and $E_2(\mathbf{p}) < 0$, then $\tanh(\frac{E_1(\mathbf{p})}{2T}) \rightarrow 1$ while $\tanh(\frac{E_2(\mathbf{p})}{2T}) \rightarrow -1$, and the integrand vanishes. The same result holds if $E_1(\mathbf{p}) < 0$ and $E_2(\mathbf{p}) > 0$. If, however, $E_1(\mathbf{p}) > 0$ and $E_2(\mathbf{p}) > 0$,

$$\frac{1}{2} \left[\tanh \left(\frac{E_1(\mathbf{p})}{2T} \right) + \tanh \left(\frac{E_2(\mathbf{p})}{2T} \right) \right] \rightarrow 1$$

reproducing the zero temperature gap equation Eq. (25). Note that even if our goal were just to understand physics at $T = 0$ it may be of practical value to do calculations at several nonzero values of the temperature and then extrapolate to $T = 0$. The reason is that at $T = 0$, specifying the boundaries of \mathcal{P} of (26), and thus the limits of integration, can be a numerical challenge. At any nonzero temperature, instead, no limits of integration need be specified. The tanh factors impose the required limits as T gets small.

We shall present $T \neq 0$ results for parameters chosen as in Fig. 4 of Ref. [9], which we first recapitulate. We specify the four-fermion interaction by choosing the cutoff parameter,

²Note that $\frac{-1}{T(e^{z/T} + 1)}$ has simple poles with unit residue at $z = i\omega_n$.

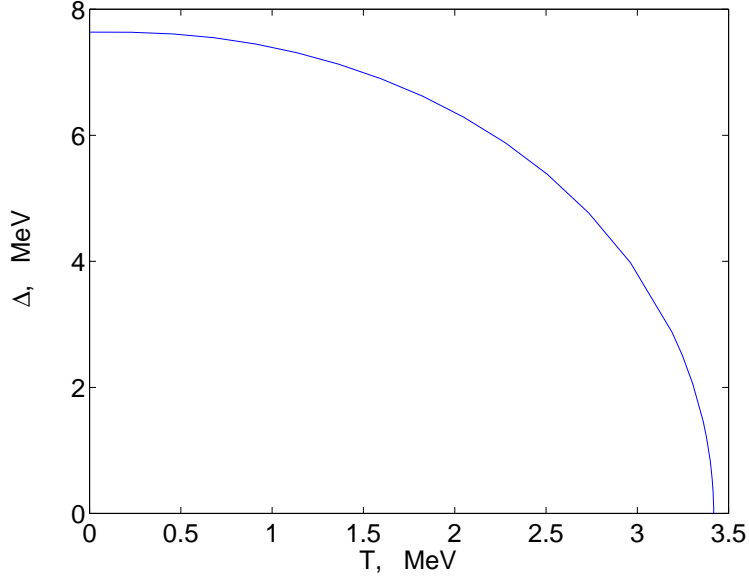


FIG. 1. The gap Δ as a function of temperature T , at $\delta\mu = \delta\mu_1$. At zero temperature, $\Delta = 7.8 \text{ MeV} = 0.195\Delta_0$. The gap vanishes above $T_c = 3.42 \text{ MeV}$.

defined in Ref. [9], to be $\Lambda = 1 \text{ GeV}$ and requiring G to be such that the BCS gap is $\Delta_0 = 40 \text{ MeV}$ at $\delta\mu = 0$. We choose $\bar{\mu} = 400 \text{ MeV}$, and explore different values of $\delta\mu$. At $T = 0$ [9], we find nonzero solutions to the gap equation (25) for $\delta\mu < \delta\mu_2 = 0.744\Delta_0$. Above $\delta\mu_2$, no pairing between u and d quarks is possible. The crystalline color superconductor phase has lower free energy than the ordinary BCS phase as long as $\delta\mu > \delta\mu_1 = 0.710\Delta_0$, where a first order phase transition occurs. A precise determination of $\delta\mu_1$ requires expressions for the free energy of both phases. The free energy of the crystalline color superconductor phase could be obtained from the gap equation along the lines described in Section 4.3 of Ref. [3]. However, $\delta\mu_1$ is well approximated by the $\delta\mu$ at which the BCS and unpaired states have equal free energy, which turns out to be $0.711\Delta_0$ [9]. At $\delta\mu = \delta\mu_1$, $\Delta = 7.8 \text{ MeV}$ [9]. For $\delta\mu \rightarrow \delta\mu_2$ from below, Δ vanishes like $(\delta\mu_2 - \delta\mu)^{1/2}$ [9]. The window $\delta\mu_1 < \delta\mu < \delta\mu_2$ widens if the interaction includes attraction in the spin-one channel. We expect this window to widen at asymptotic density, where quarks interact by exchanging a propagating gluon.

In Fig. 1, we show the dependence of Δ on the temperature T at $\delta\mu = \delta\mu_1$. We find that the critical temperature above which the crystalline color superconductivity is lost is $T_c = 3.42 \text{ MeV}$, corresponding to $T_c = 0.44\Delta(T = 0)$. In Fig. 2, we plot both T_c and $\Delta(T = 0)$ as functions of $\delta\mu$, for $\delta\mu_1 < \delta\mu < \delta\mu_2$. We find that the ratio $T_c/\Delta(T = 0)$ changes little, decreasing from 0.44 at $\delta\mu_1$ to 0.39 for $\delta\mu \rightarrow \delta\mu_2$. This agrees with the previously known result that $T_c/\Delta(T = 0) \rightarrow \sqrt{3}/2\pi^2$ for $\delta\mu \rightarrow \delta\mu_2$ [19]. There are two *a priori* reasons why one may have questioned whether T_c in the crystalline color superconducting phase would turn out to be proportional to $\Delta(T = 0)$. First, this phase is in fact gapless. There are directions in momentum space (which intersect the boundaries of the blocking regions) for which gapless excitations exist at zero temperature. One may wonder whether the presence of these gapless modes, which can be excited at arbitrarily low temperature, might lower T_c . Second, the condensation energy in the crystalline color superconductor phase is of order

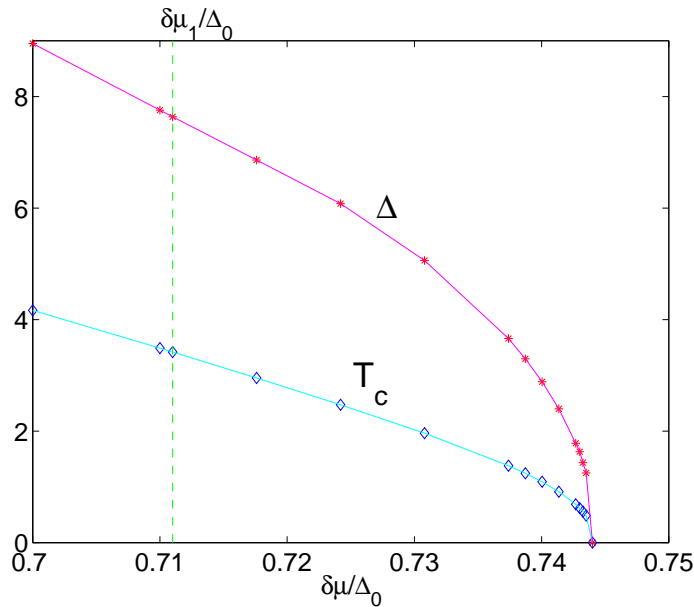


FIG. 2. The zero temperature gap Δ and the critical temperature T_c (both in MeV) as functions of $\delta\mu$. Δ vanishes like $(\delta\mu_2 - \delta\mu)^{1/2}$ for $\delta\mu \rightarrow \delta\mu_2$. In this limit, $T_c/\Delta \rightarrow 0.39$. To the left of $\delta\mu_1$, the ordinary BCS phase is favored.

$\bar{\mu}^2\Delta^4/\Delta_0^2$ [9], whereas that in the ordinary BCS phase is of order $\bar{\mu}^2\Delta_0^2$. One may therefore wonder whether the T_c for crystalline color superconductivity scales differently with Δ . It turns out, however, that the simplest expectation holds true: Δ is the gap in the fermion spectrum in directions in momentum space along which pairing is maximized and destroying the condensate therefore requires a temperature T_c which is of order Δ .

ACKNOWLEDGMENTS

We acknowledge helpful discussions with M. Alford.

Research supported in part by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818. The work of KR is supported in part by a DOE OJI grant and by the Alfred P. Sloan Foundation. The work of JAB and JK is supported in part by DOD National Defense Science and Engineering Graduate Fellowships.

REFERENCES

- [1] M. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. **B537**, 443 (1999) [hep-ph/9804403]
- [2] R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, Annals Phys. **280**, 35 (2000) [hep-ph/9904353]; T. Schäfer, Nucl. Phys. **B575**, 269 (2000); [hep-ph/9909574]; I. A. Shovkovy and L. C. Wijewardhana, Phys. Lett. **B470**, 189 (1999) [hep-ph/9910225]; N. Evans, J. Hormuzdiar, S. D. Hsu and M. Schwetz, Nucl. Phys. **B581**, 391 (2000) [hep-ph/9910313].
- [3] For a review, see K. Rajagopal and F. Wilczek, to appear in B. L. Ioffe Festschrift, “At the Frontier of Particle Physics/Handbook of QCD”, M. Shifman, ed., (World Scientific, 2001) [hep-ph/0011333].
- [4] M. Alford, J. Berges and K. Rajagopal, Nucl. Phys. **B558**, 219 (1999) [hep-ph/9903502].
- [5] T. Schäfer and F. Wilczek, Phys. Rev. **D60**, 074014 (1999) [hep-ph/9903503].
- [6] K. Rajagopal and F. Wilczek, hep-ph/0012039.
- [7] A. M. Clogston, Phys. Rev. Lett. **9**, 266 (1962); B. S. Chandrasekhar, App. Phys. Lett. **1**, 7 (1962).
- [8] P. F. Bedaque, hep-ph/9910247.
- [9] M. Alford, J. Bowers and K. Rajagopal, to appear in Phys. Rev. D, hep-ph/0008208.
- [10] B. Barrois, Nucl. Phys. **B129**, 390 (1977); S. Frautschi, Proceedings of workshop on hadronic matter at extreme density, Erice 1978; B. Barrois, “Nonperturbative effects in dense quark matter”, Cal Tech PhD thesis, UMI 79-04847-mc (1979).
- [11] D. Bailin and A. Love, Phys. Rept. **107**, 325 (1984), and references therein.
- [12] M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. **B422**, 247 (1998) [hep-ph/9711395].
- [13] R. Rapp, T. Schäfer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998) [hep-ph/9711396].
- [14] T. Schaefer, Phys. Rev. **D62**, 094007 (2000) [hep-ph/0006034].
- [15] A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
- [16] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).
- [17] D. T. Son and M. A. Stephanov, hep-ph/0005225; K. Splittorff, D. T. Son and M. A. Stephanov, hep-ph/0012274.
- [18] A. Sedrakian, nucl-th/0008052. The related unpairing transition was discussed in the absence of LOFF pairing in A. Sedrakian and U. Lombardo, Phys. Rev. Lett. **84**, 602 (2000).
- [19] S. Takada and T. Izuyama, Prog. Theor. Phys. **41**, 635 (1969).
- [20] D. T. Son, Phys. Rev. **D59**, 094019 (1999) [hep-ph/9812287].
- [21] K. Rajagopal and E. Shuster, Phys. Rev. D **62**, 085007 (2000) [hep-ph/0004074].
- [22] R. Rapp, E. Shuryak and I. Zahed, hep-ph/0008207.
- [23] R. D. Pisarski and D. H. Rischke, Phys. Rev. **D60**, 094013 (1999) [nucl-th/9903023]; Phys. Rev. **D61**, 051501 (2000) [nucl-th/9907041]; Phys. Rev. **D61**, 074017 (2000) [nucl-th/9910056].
- [24] T. Schäfer and F. Wilczek, Phys. Rev. **D60**, 114033 (1999) [hep-ph/9906512].